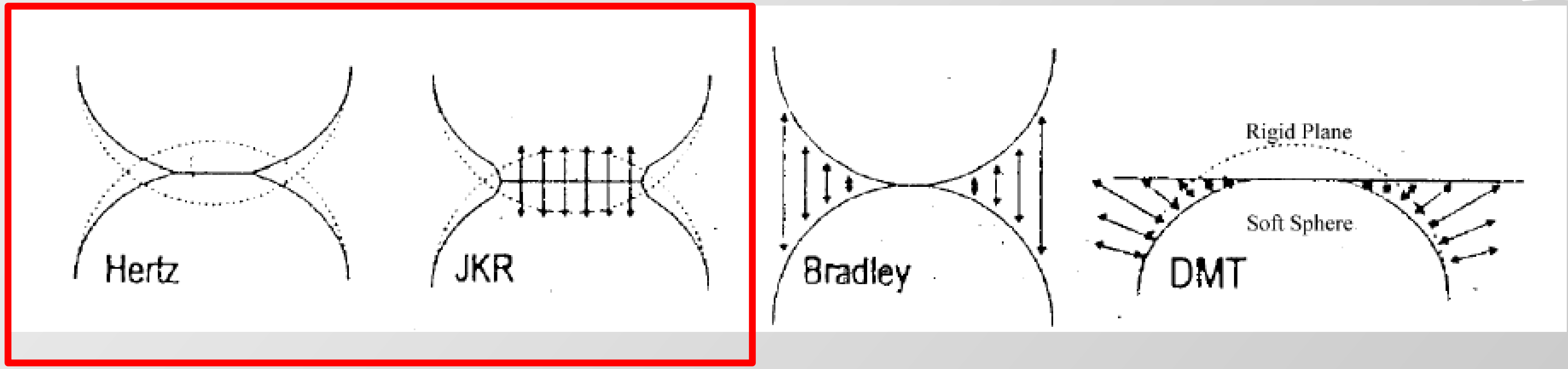
2. Basic Indentation Theory

Andy Bushby

Contact mechanics models

Contact between two elastic spheres



Hertz	1880	Elastic contact only
(**JKR**) Johnson, Kendall and Roberts	1971	Elastic with adhesion
Bradley	1932	van der Waals forces only
(**DMT**), Derjaguin, Muller, Toporov and Yu	1975	Elastic, adhesion and vdW

These are only **elastic and surface force theories**;
none of them consider plastic deformation
or time dependent deformation at the contact

J.A. Greenwood Proc. Roy. Soc, London A453 1277 (1997)

Elastic contact mechanics: Love Hertz

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Heinrich Hertz

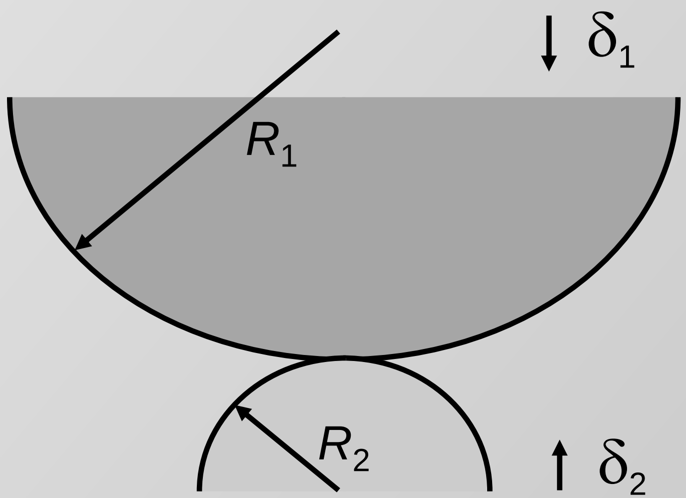
- 1857-1894
- PhD in optical properties at age of 23
- Became assistant to Helmholtz
- Major contribution to existence and properties of electro-magnetic waves
- Unit of frequency named after him, Hz
- Died of blood poisoning at the age of 36

While working on optical properties with compound lenses he noticed that the glass lenses distorted at the contact between them.
This messed-up his experiments, so he thought about how the distortion occurred, using the (then new) theory of elastic mechanics.
Devises theory of elastic contact mechanics during the Christmas holidays at age of 23

Elastic contact mechanics

Hertzian contact mechanics - 1880

Hertz considered the elastic contact between any 2 non-conforming bodies



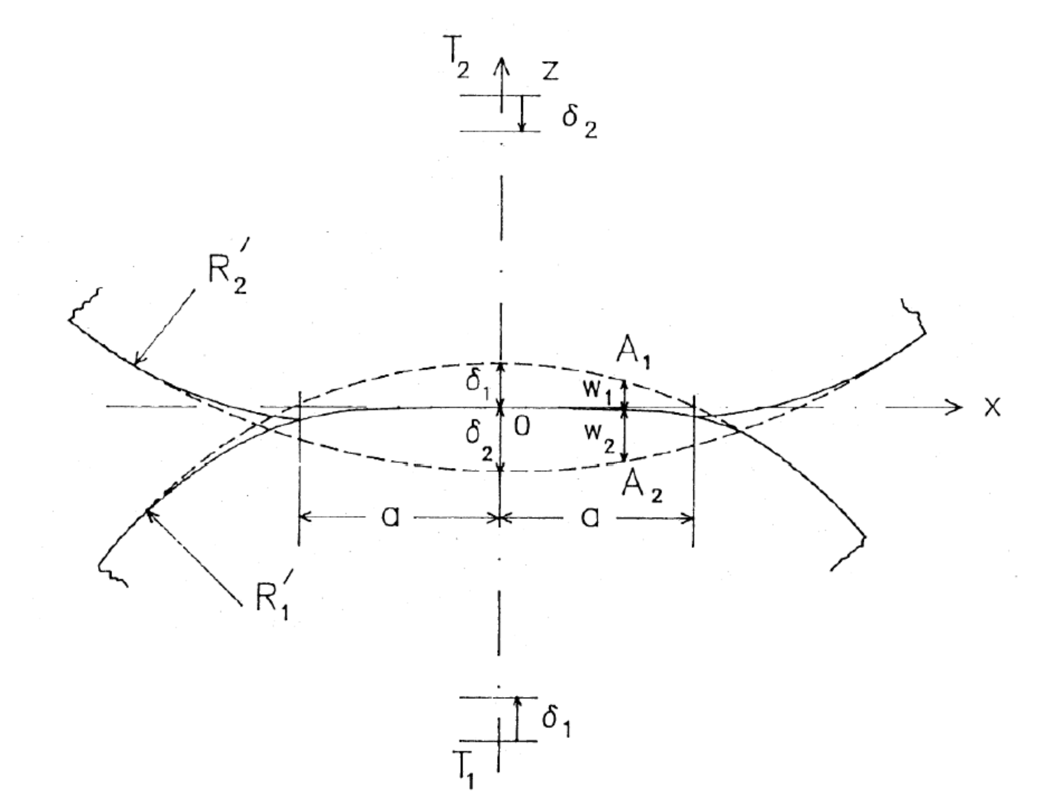
Assumptions:

- The material is isotropic and homogenous (e.g. glass)
- Strains are small and below the elastic limit (i.e. linear elastic everywhere)
- Each solid can be considered as an elastic half space (i.e. infinitely large)
- The surfaces are continuous and non-conforming (i.e. only touch at 1 place)
- The contact is frictionless (i.e. no friction to complicate the displacements)

Elastic contact mechanics

From consideration of the elastic displacements, *w*, at the contact Hertz calculated the approach of distant points in the 2 bodies as

$$\delta = \delta_1 + \delta_2$$



The 2 bodies in contact form a contact area over which force balance is maintained (i.e. the reaction forces of both bodies are in equilibrium – resulting from their elastic properties). The contact area has radius '*a*'.

Elastic contact mechanics

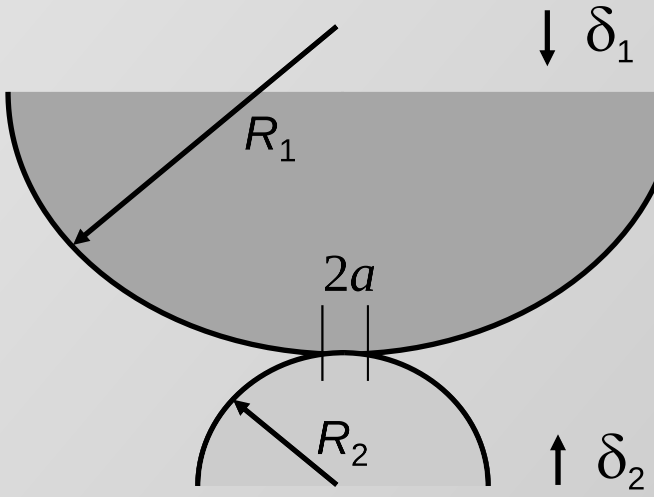
Hertz derived a series of important relations

For the **total displacement** δ

$$\delta_1 + \delta_2 = \delta = \left(\frac{9}{16} \frac{F^2}{E^{*2}} \frac{1}{R^*} \right)^{1/3}$$

and the **radius of the contact** area, *a*,

$$a^2 = \delta R^* = \left(\frac{3}{4} \frac{FR^*}{E^*} \right)^{2/3}$$



where *E** is the combined elastic moduli of the 2 materials

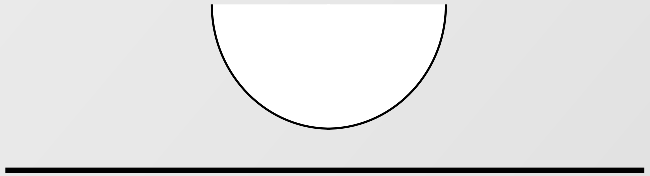
$$\frac{1}{E^*} = \left(\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right)$$

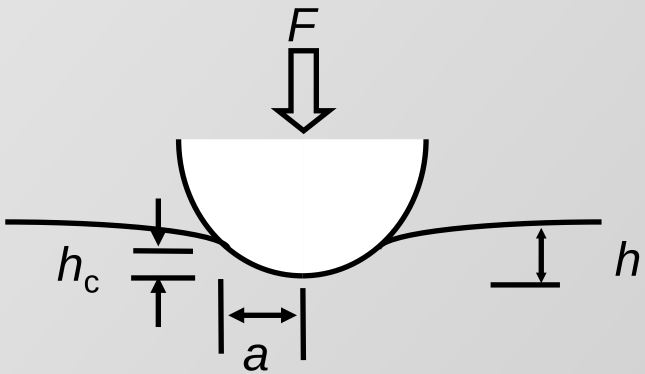
*R** is the relative curvature between the 2 surfaces

$$\frac{1}{R^*} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Elastic contact mechanics

Indentation is a special case of Hertzian mechanics where $R_2 = \infty$





$$h = \left(\frac{9}{16} \frac{F^2}{E^{*2}} \frac{1}{R} \right)^{1/3}$$

$$h_c = \frac{h_e}{2}$$

Contact depth = ½ total depth

$$a^2 = \left(2Rh_c - h_c^2 \right)$$

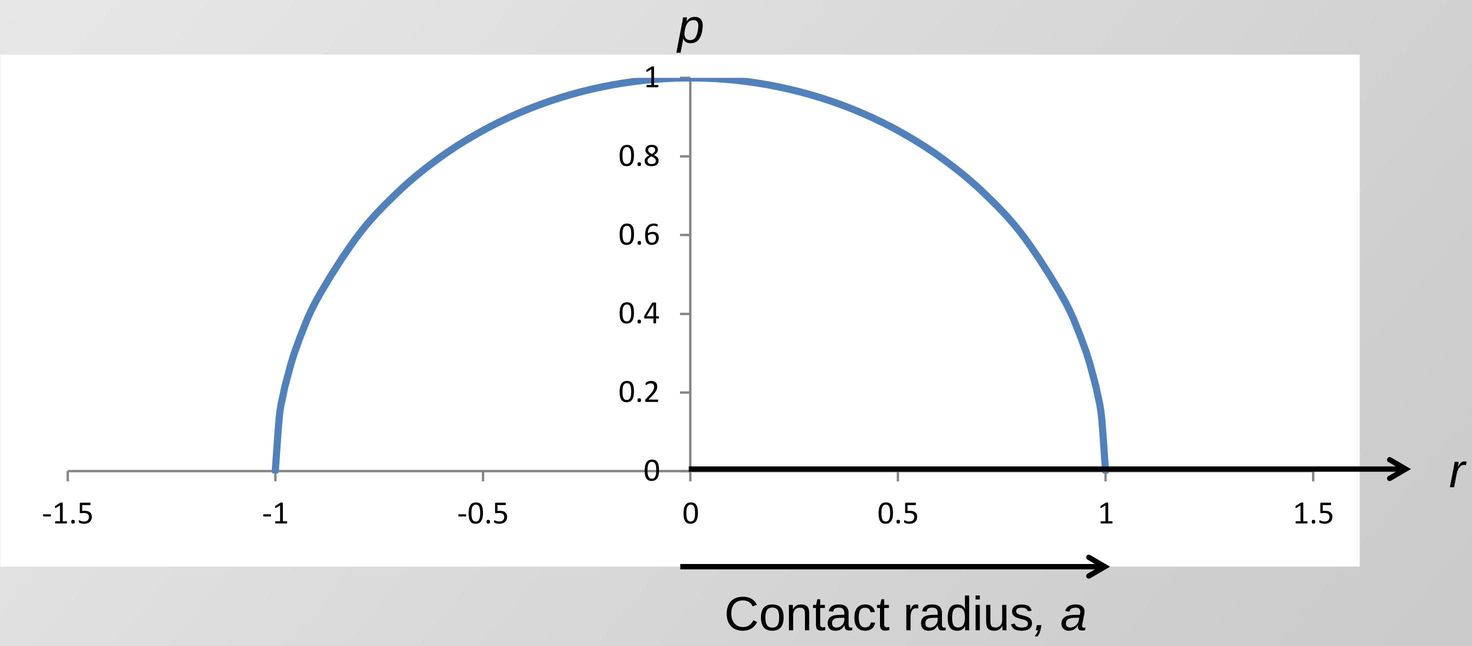
$$\frac{1}{E^*} = \left(\frac{\left(1 - \nu_m^2\right)}{E_m} + \frac{\left(1 - \nu_i^2\right)}{E_i} \right)$$

Subscripts *m* = material, *i* = indenter

Hertzian pressure distribution

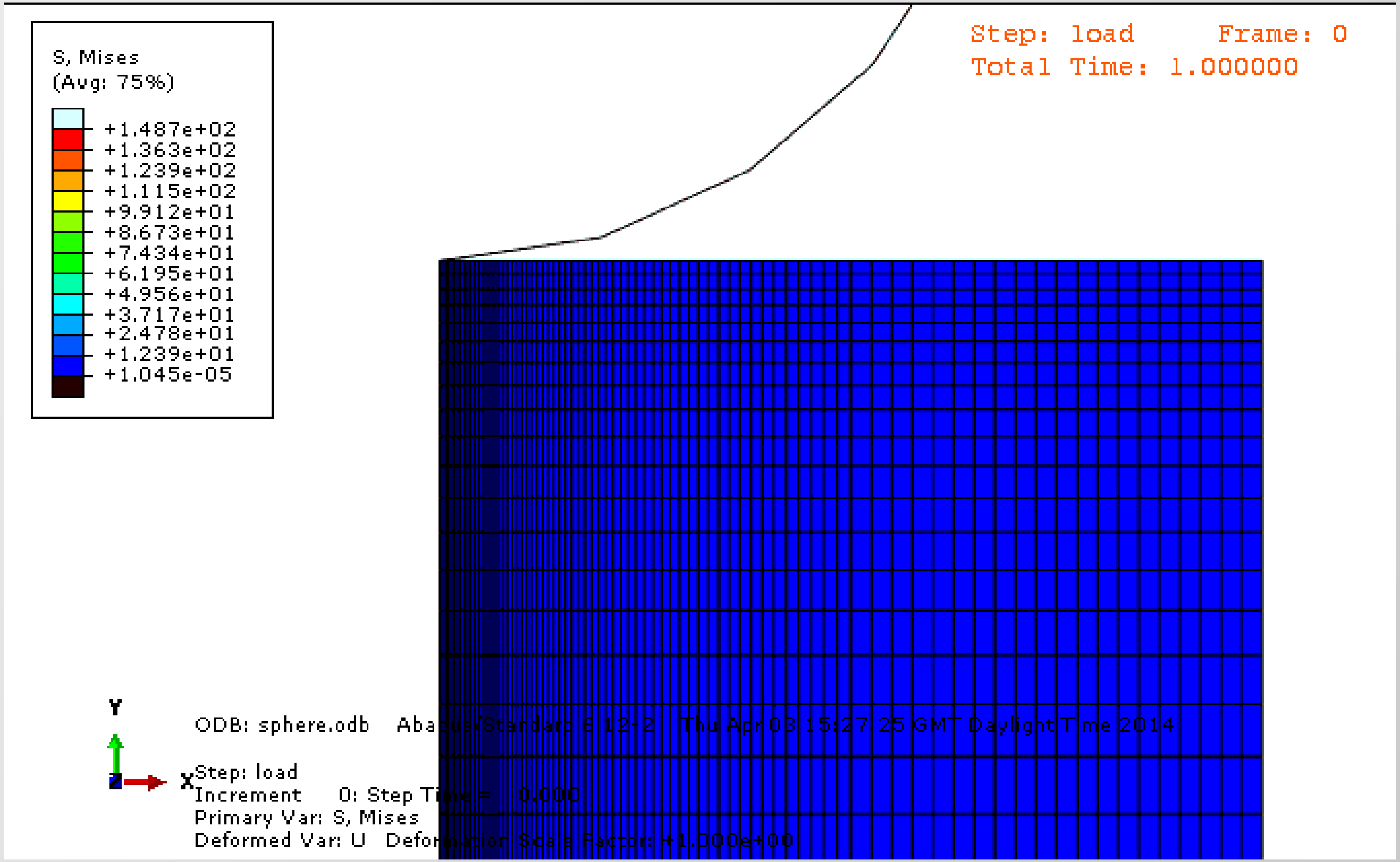
For contacting spheres the maximum pressure is $p_0 = \frac{2F}{\pi a} = \left(\frac{FE^*}{\pi R^*}\right)^{1/2}$

And the pressure distribution is $p = p_0(1 - r^2/a^2)^{1/2}$



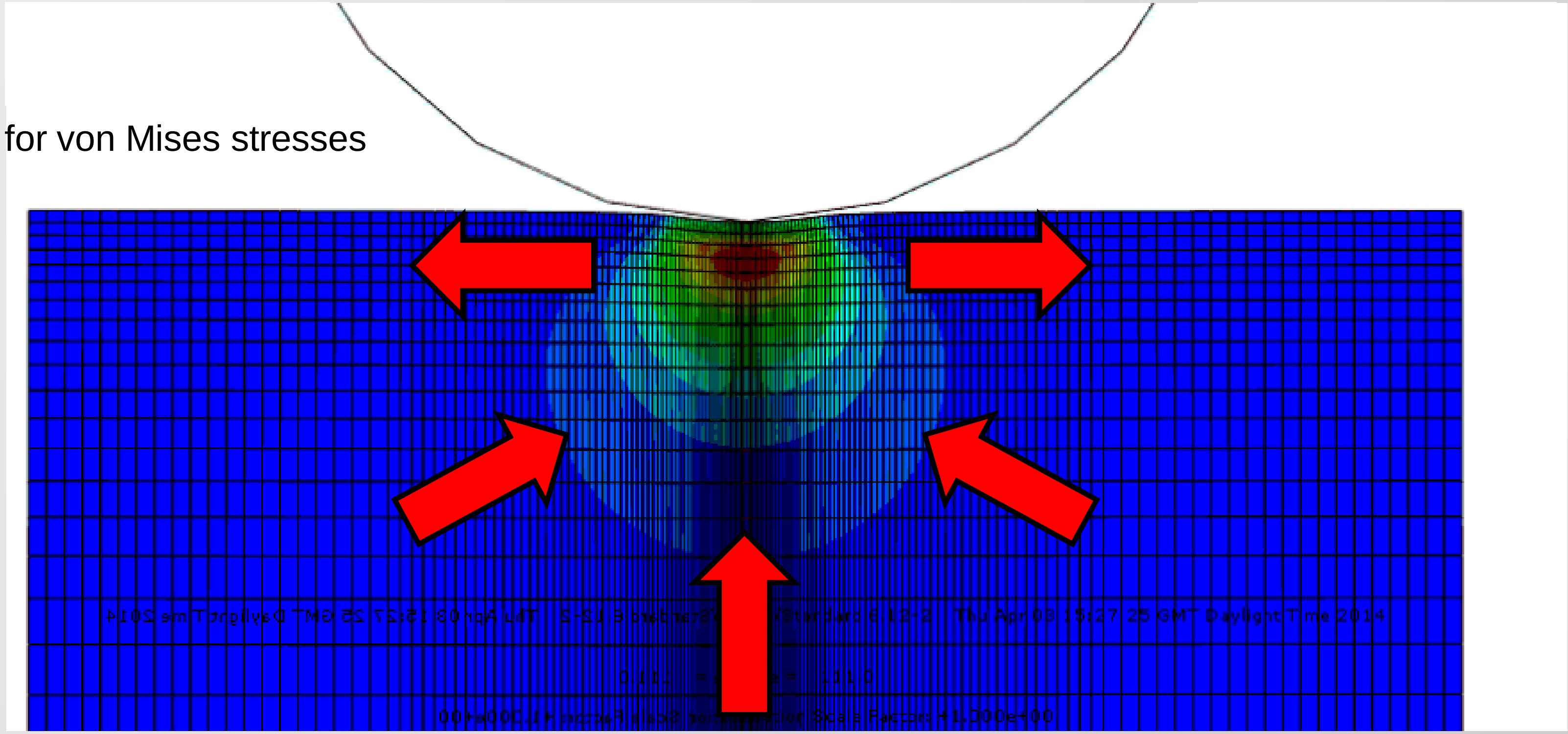
where *F* = applied force, *E** and *R** defined on slide 12

Contact stresses



Contact stresses

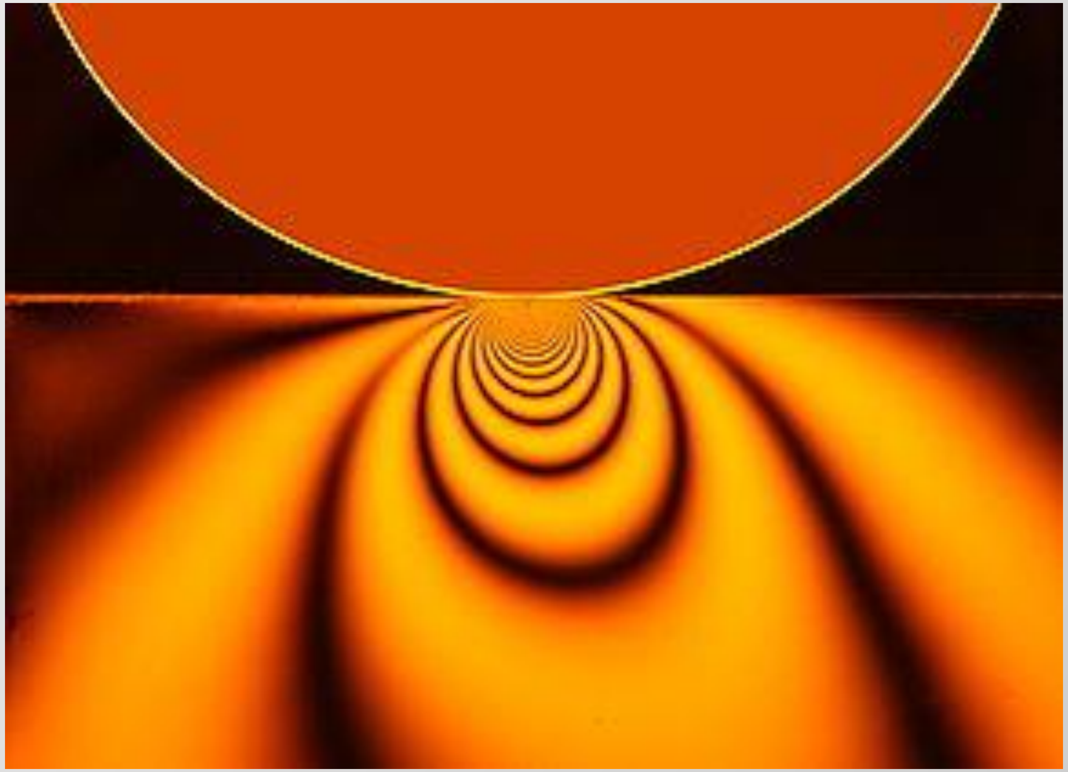
FEA simulation for von Mises stresses



Radial stress fields falling off as $1/r^2$
Tensile, compressive and shear components
Constraint - large hydrostatic component beneath contact

Sharper the contact – more intense the stress concentration
$\Rightarrow$ plasticity or cracking

Contact stresses



Contours of
shear stress

(Photo-elastic effect)

Radial stress fields falling off as $1/r^2$
Tensile, compressive and shear components
Constraint - large hydrostatic component beneath contact

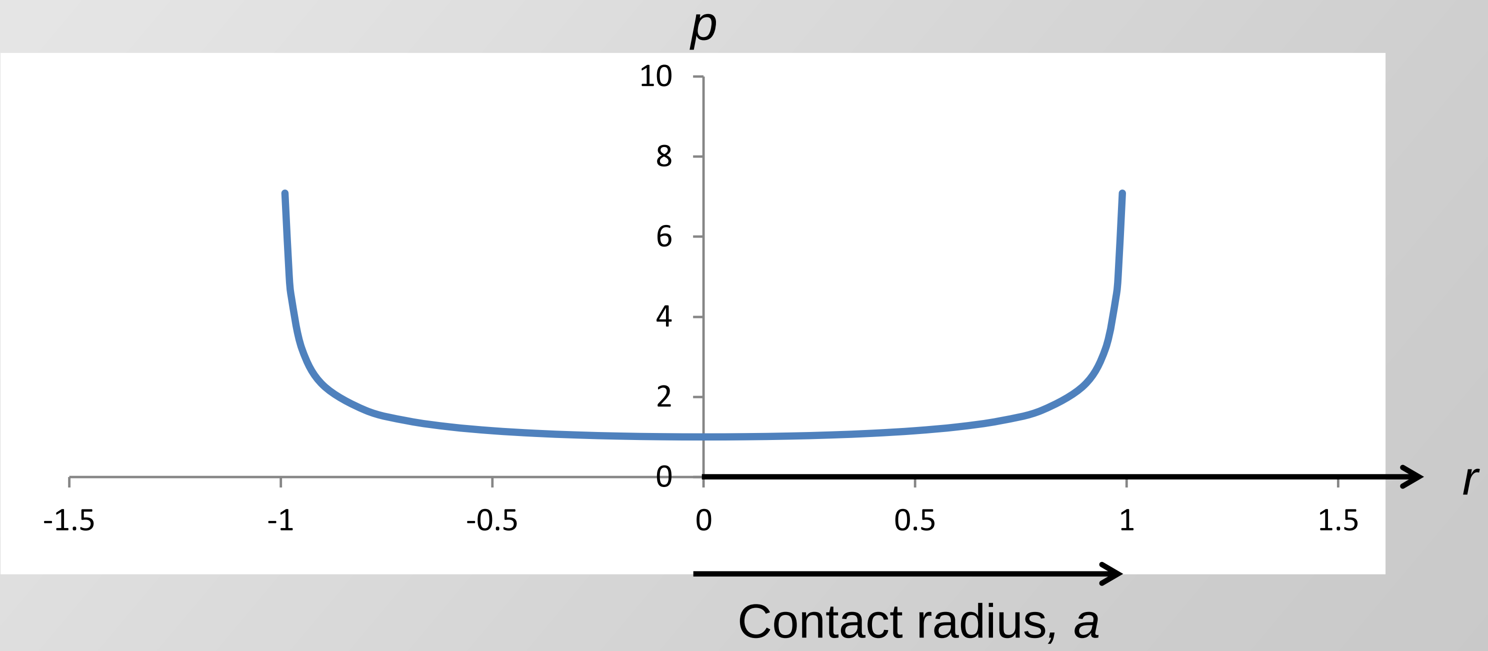
Sharper the contact – more intense the stress concentration
⇒ plasticity or cracking

Pressure distribution - punch

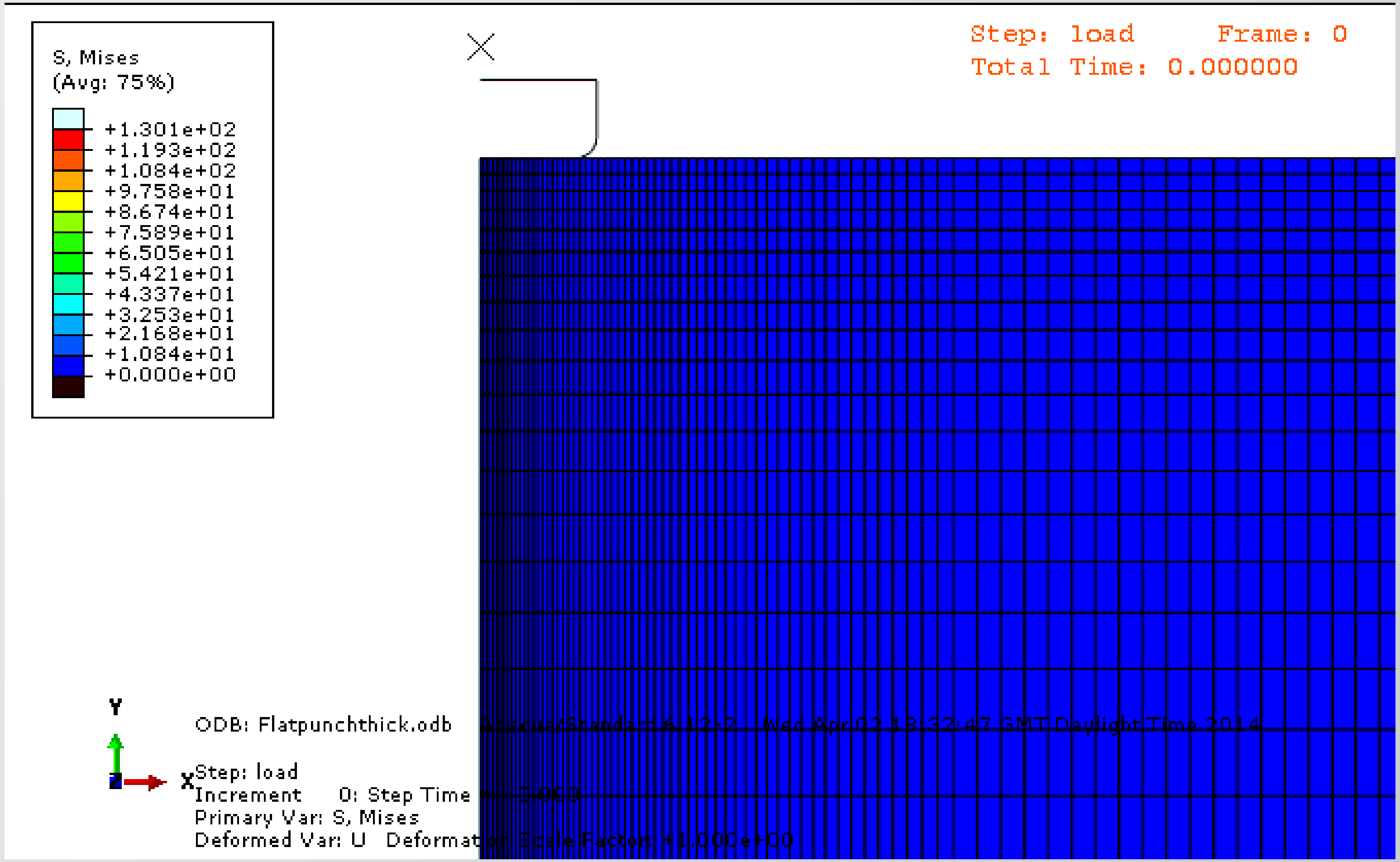
Pressure distribution for a flat punch indenter

The maximum pressure is at the edge of contact

And the pressure distribution is $p = p_0(1 - r^2/a^2)^{-1/2}$



Contact stresses

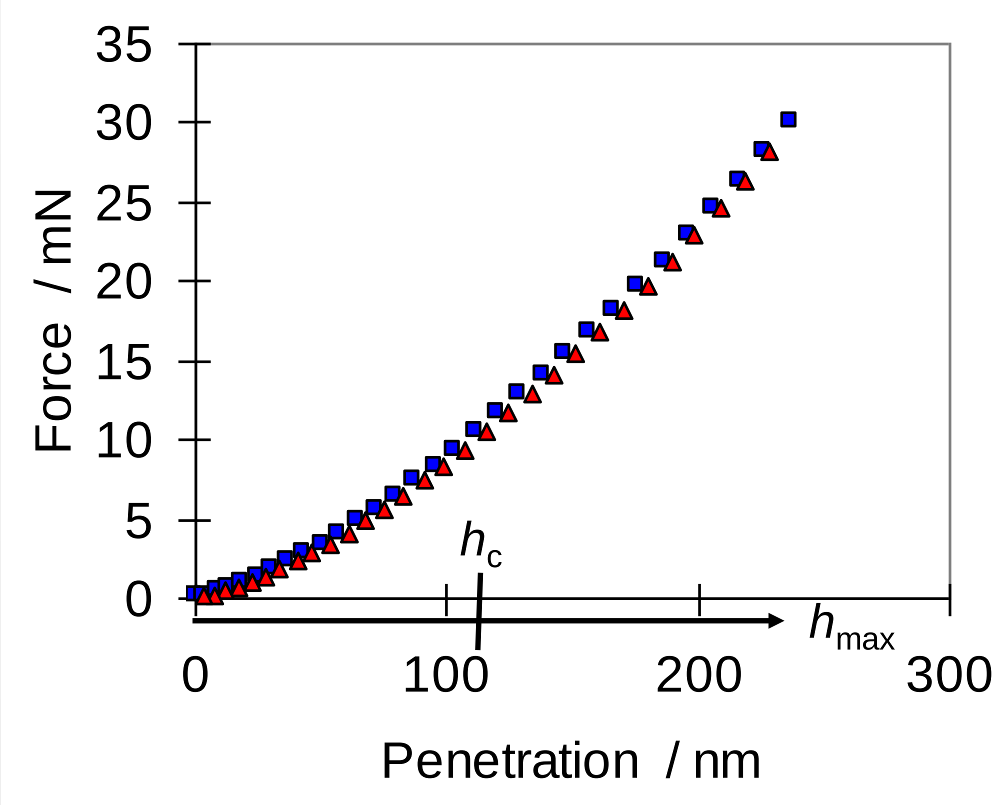


Elastic (Hertzian) response - reversible

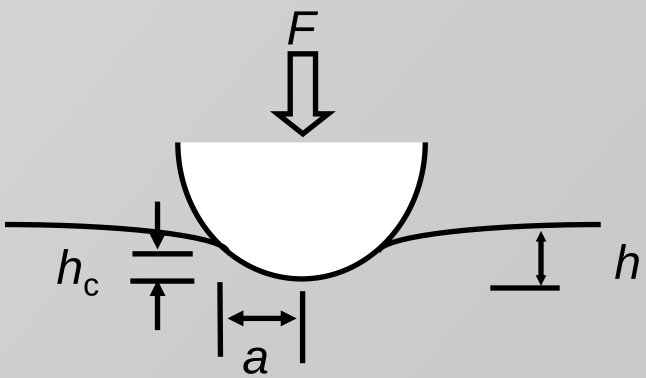
Sneddon, 1965

$$F = \varepsilon h^m$$

m = 1 (flat punch)
m = 2 (cone)
m = 1.5 (sphere)
m = 1.5 (paraboloid)



$$h \propto F^{2/3}$$



$$h_c = \frac{h_e}{2}$$

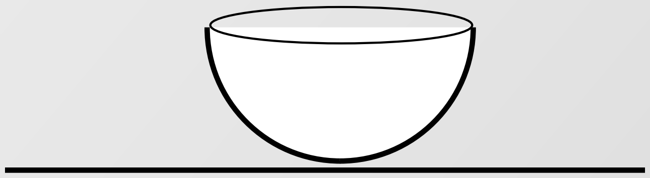
Half of elastic displacement

Elastic modulus $E^* = \left(\frac{3}{4}\frac{F}{h^{3/2}}\frac{1}{R^{1/2}}\right)$

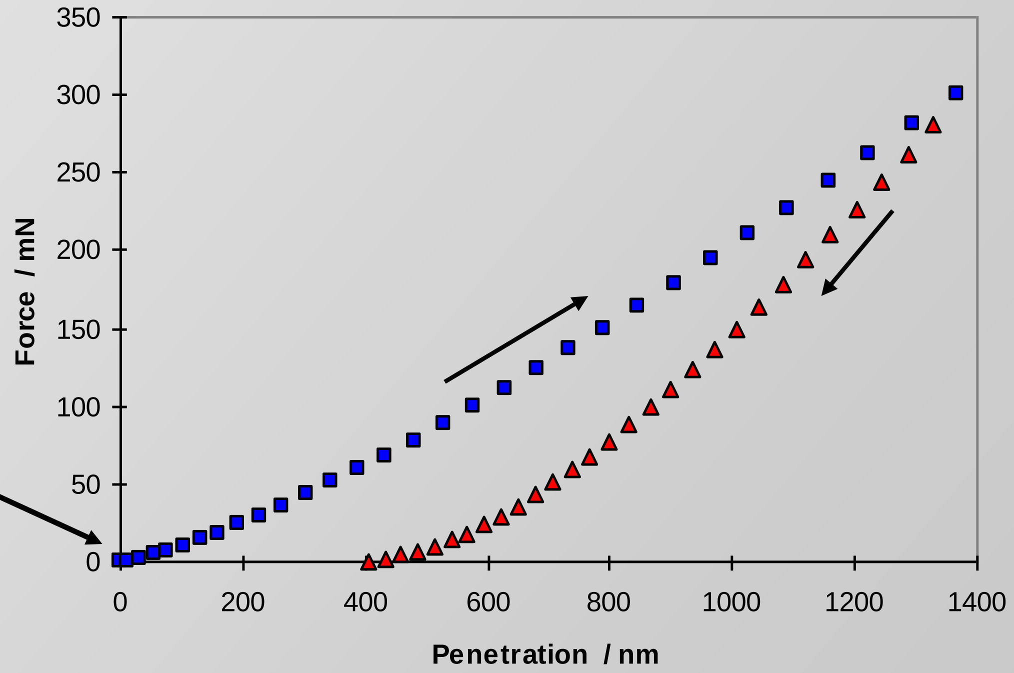
I.N.Sneddon Int. J. Eng. Sci. Vol 3 p47 (1965)

Elastic-plastic contact

Typical nanoindentation experiment
Example for a spherical indenter on glass

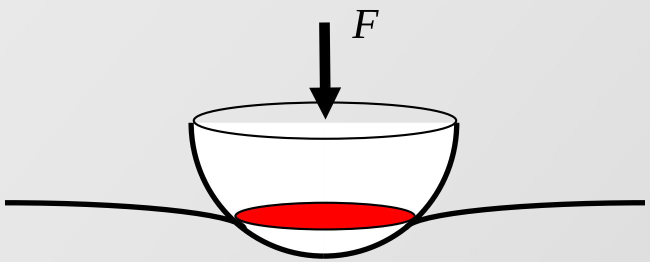


Approach to contact with surface

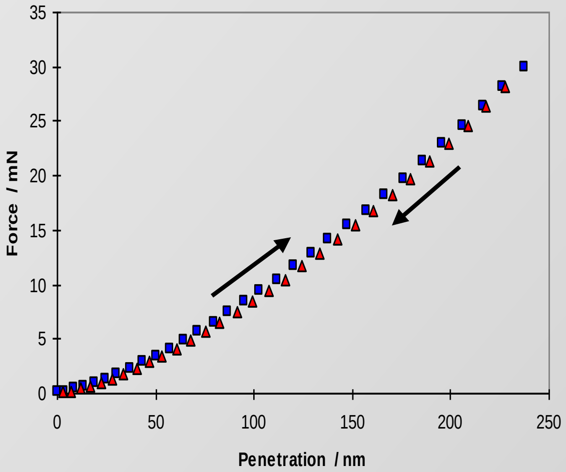


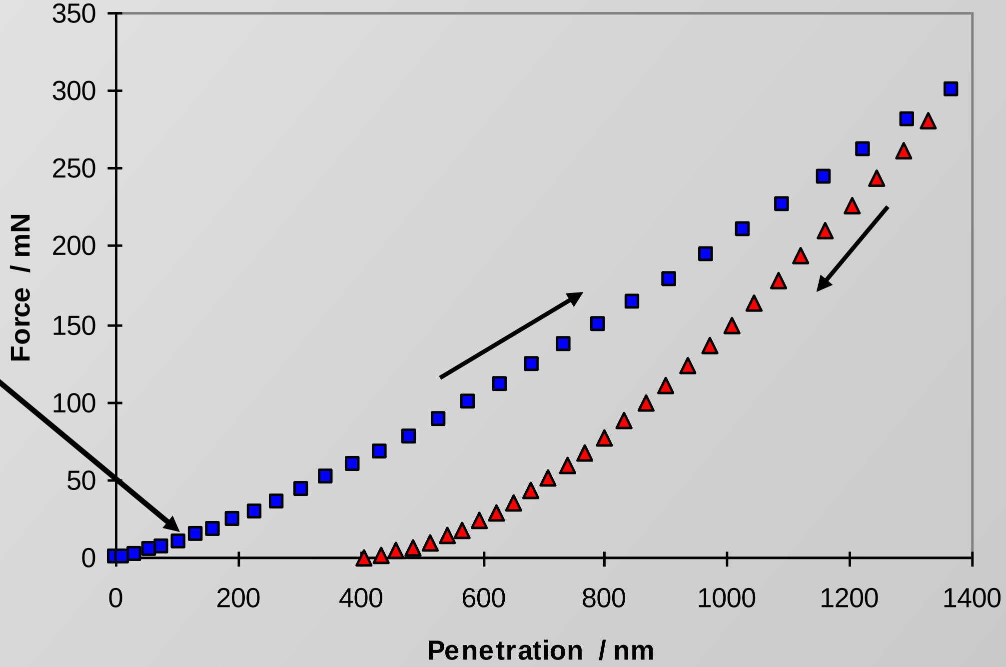
Elastic-plastic contact

Typical nanoindentation experiment
Example for a spherical indenter on glass



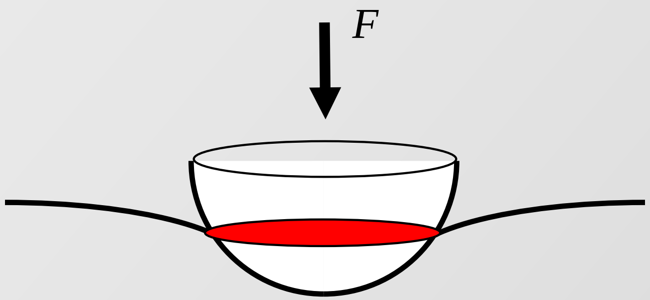
Initial elastic response



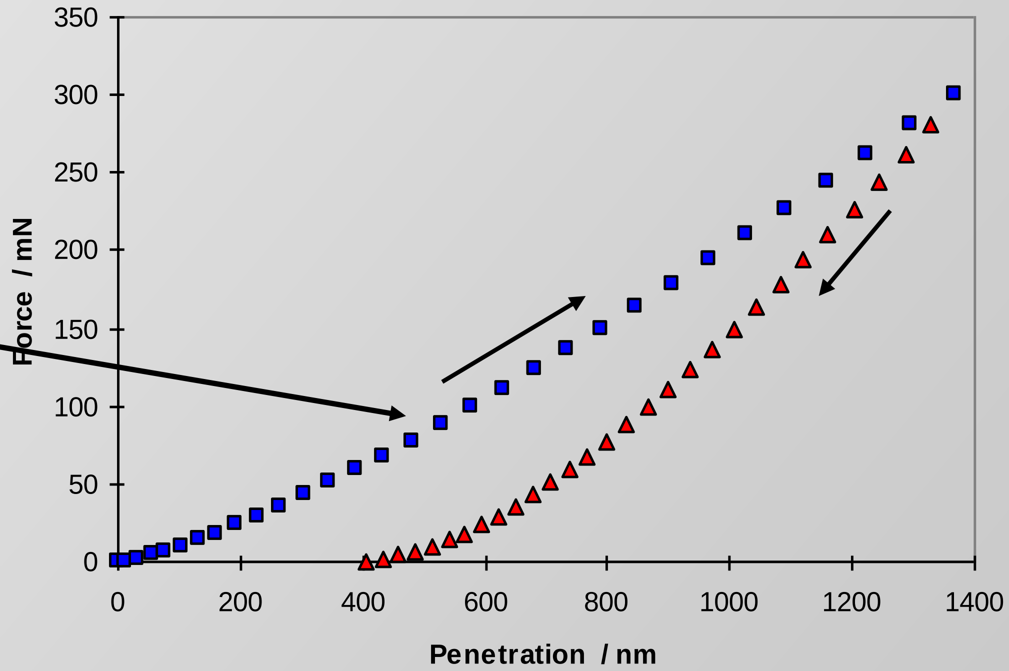


Elastic-plastic contact

Typical nanoindentation experiment
Example for a spherical indenter on glass



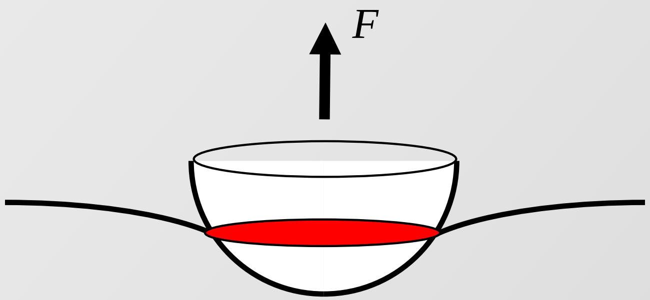
Elastic + plastic response
contact area increasing



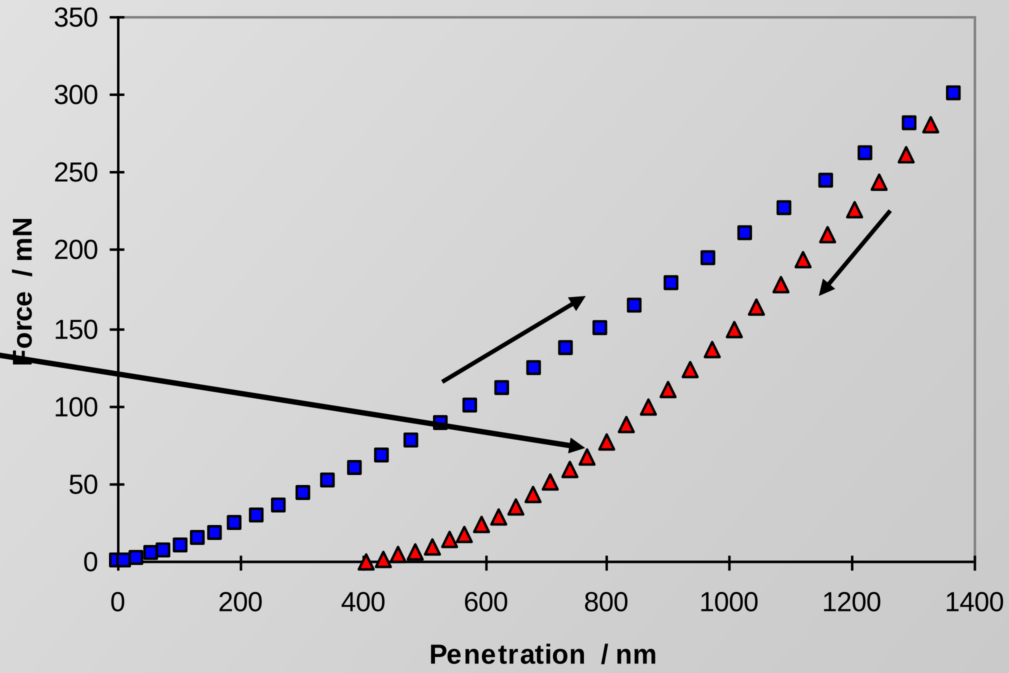
Contact pressures are large → most materials permanently deform

Elastic-plastic contact

Typical nanoindentation experiment
Example for a spherical indenter on glass

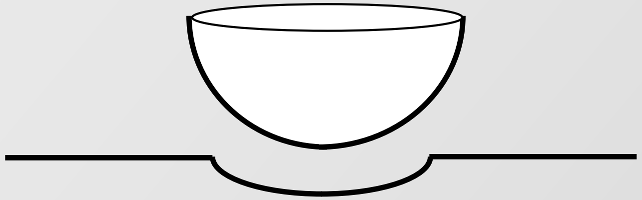


Elastic recovery
contact area decreasing

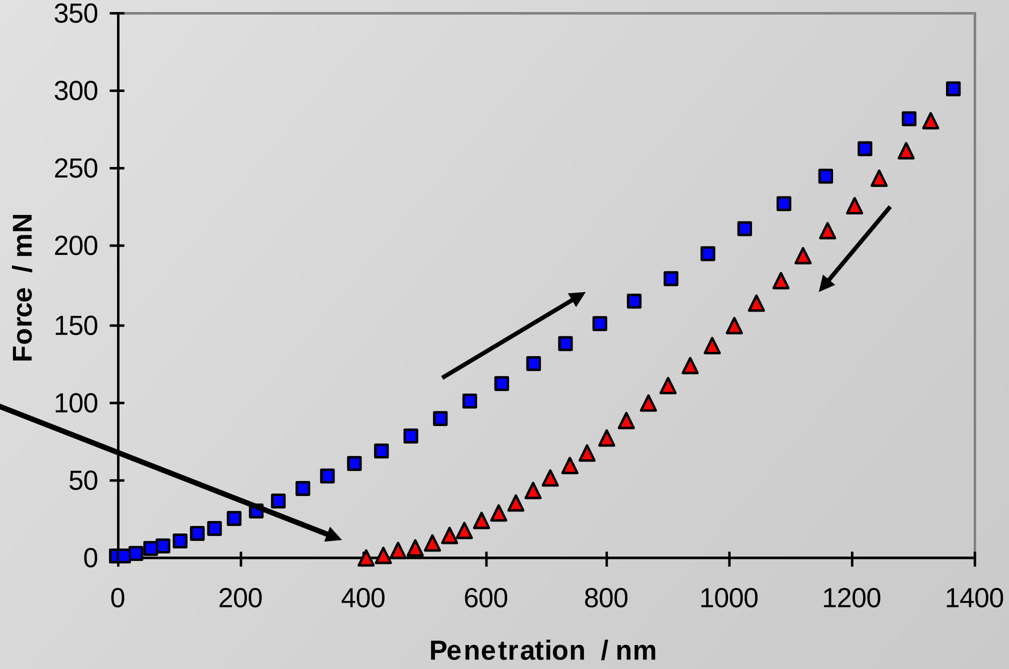


Elastic-plastic contact

Typical nanoindentation experiment
Example for a spherical indenter on glass



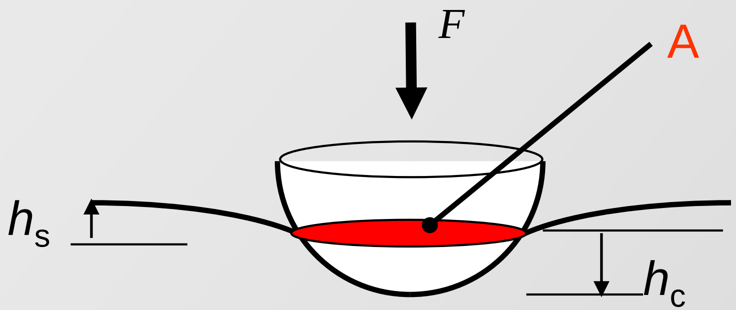
Residual impression



= permanent deformation

Elastic-plastic contact

Unload is assumed to be purely elastic recovery



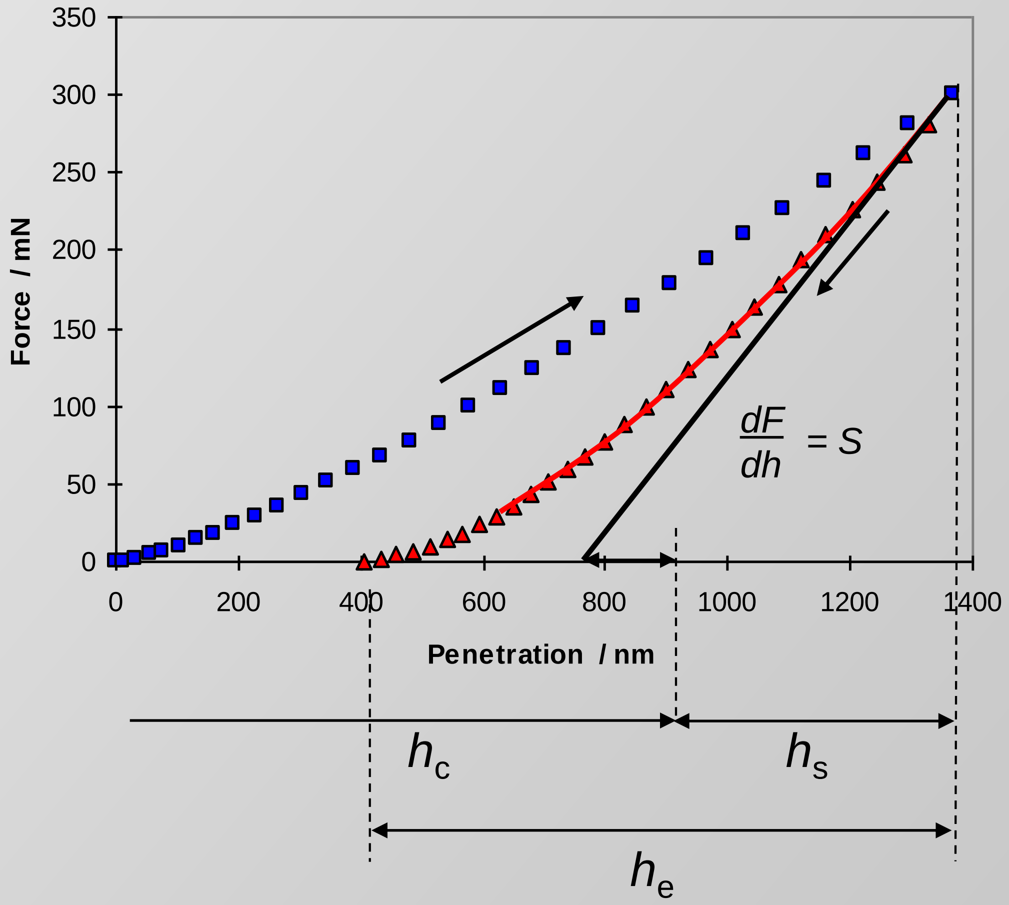
$$h_c = h_{max} - \varepsilon \frac{F_{max}}{S}$$

$\varepsilon = 1$ (flat punch)

$\varepsilon = 0.72$ (cone)

$\varepsilon = 0.75$ (sphere)

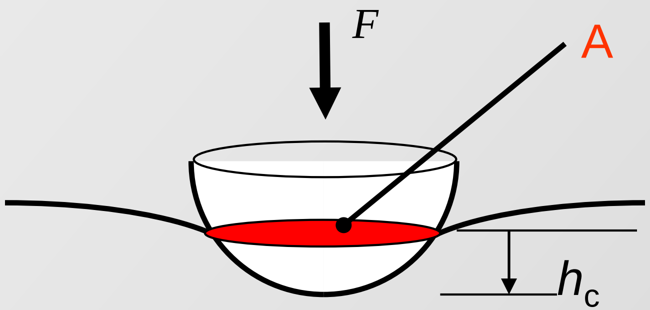
$\varepsilon = 0.75$ (paraboloid)

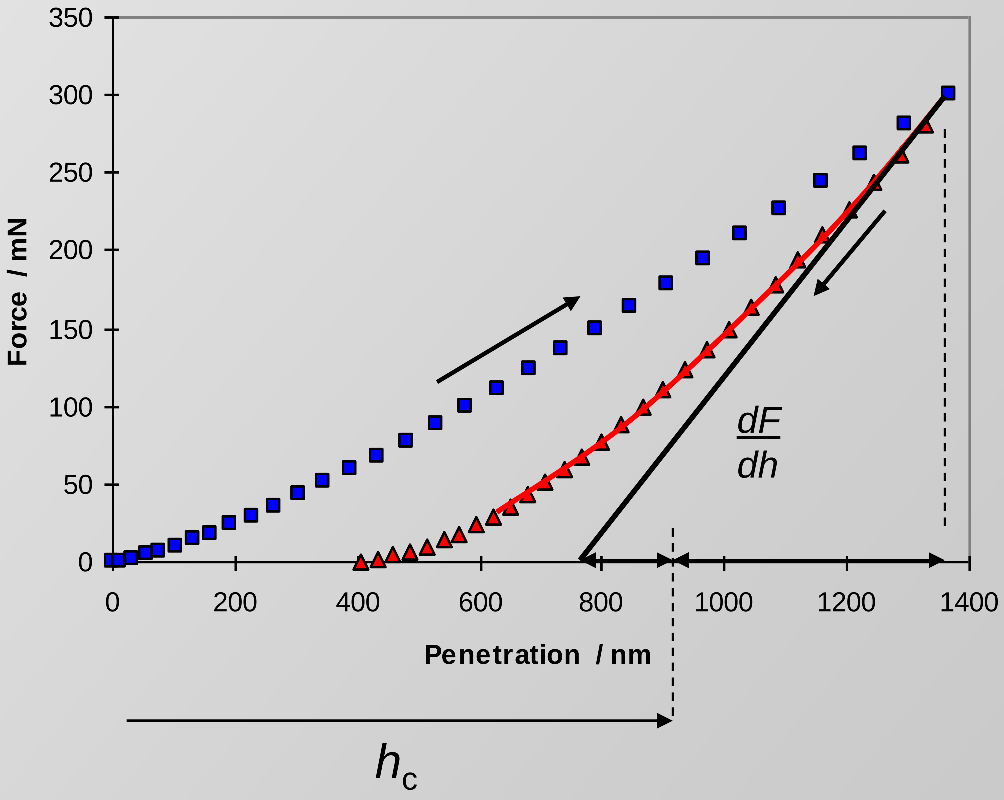


(from elastic recovery rate – Sneddon, 1965)

Elastic-plastic contact

Unload is assumed to be purely elastic recovery





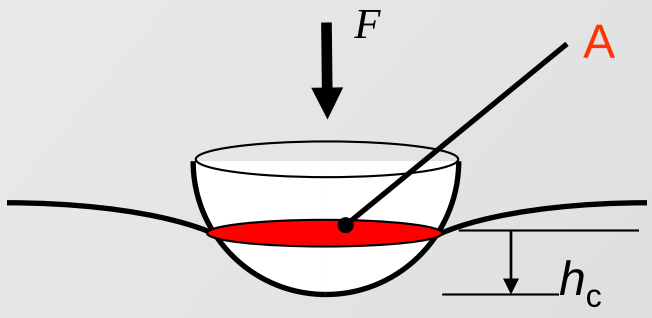
Determine contact depth, *hc*, and hence contact area, *A* from the tip geometry

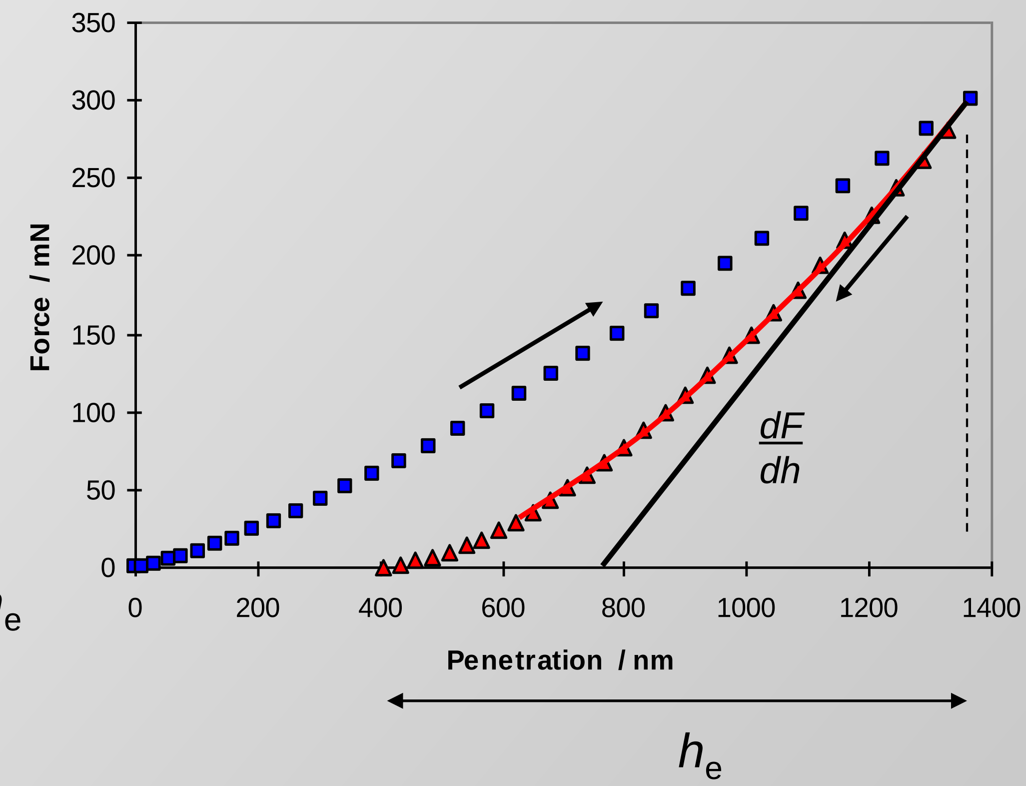
$$h_e = 2(h_{max} - h_c)$$ (Hertz)

$$h_c = h_{max} - 0.75\, F \frac{dh}{dF}$$ (from elastic recovery rate – Sneddon, 1965)

Elastic-plastic contact

Calculate material properties from unloading data





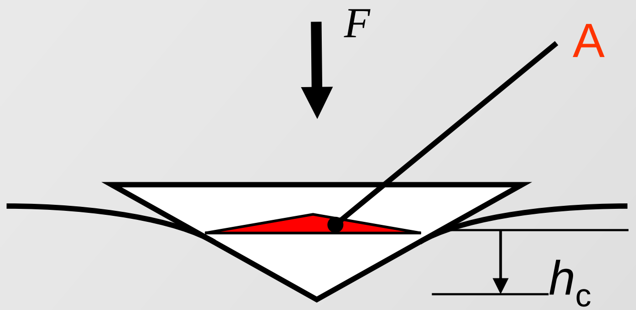
Determine contact radius, *a*
and hence elastic modulus
from the elastic displacement, h_e

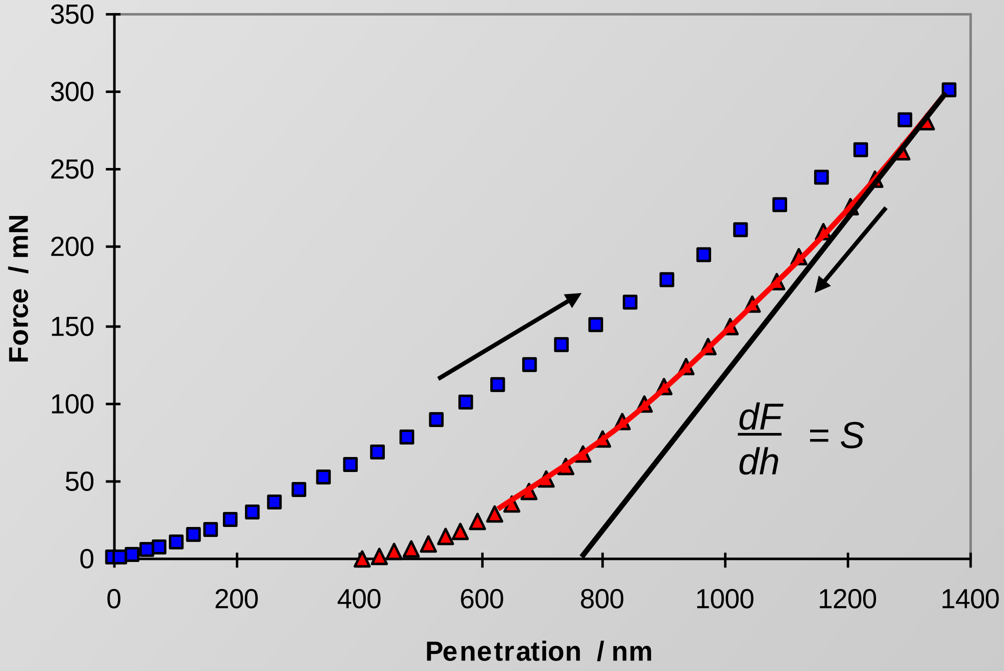
$$a^2 = \left(2Rh_c - h_c^2\right)$$

$$E^* = \left(\frac{3}{4}\,\frac{F}{h_e^{3/2}}\,\frac{1}{R^{*1/2}}\right)$$

Oliver and Pharr method

Calculate material properties from unloading data (for Berkovich indenters)





Hardness $H = \frac{F}{A}$

Elastic modulus $E^* = \frac{1}{\beta}\frac{\sqrt{\pi}}{2}\frac{S}{\sqrt{A}}$

where β is a factor to account for the corners of the Berkovich indenter (β ≈ 1.034)

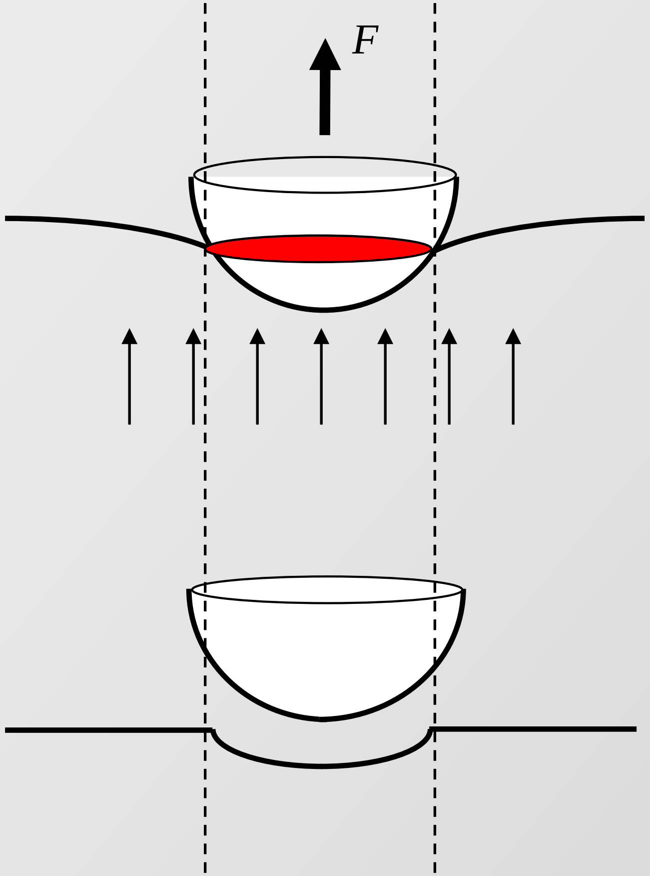
and *S = dF/dh* the contact stiffness

OK for elastic-plastic materials (i.e. metals and ceramics) but not anything else!

W.C. Oliver and G.M. Pharr, Journal of Materials Research Vol. 7 p1564 (1992)

Elastic recovery

Elastic recovery assumed to be vertical
so that recovered impression size = contact size

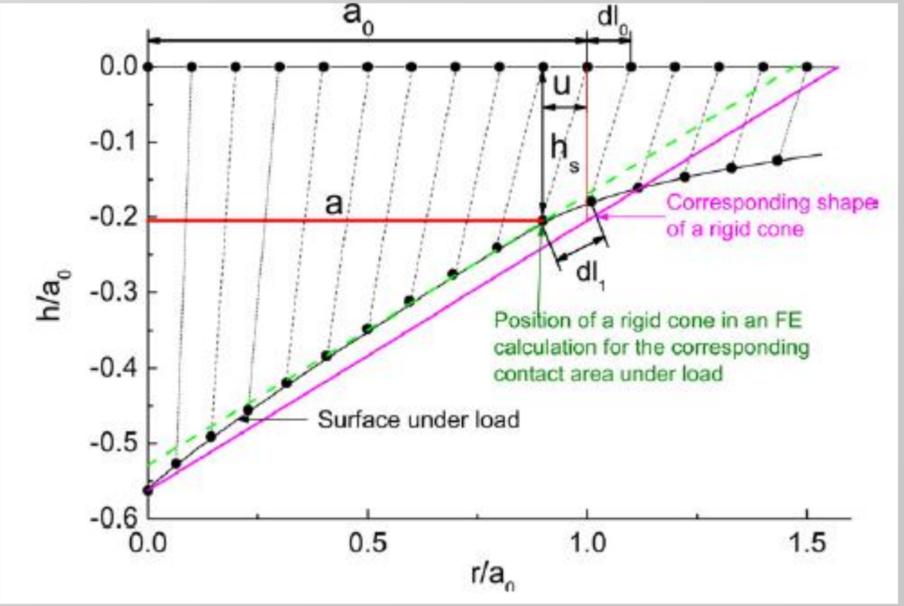


Lateral contraction depends on E/H and ν
Significant for low E/H materials such as fused silica

The contact radius under load, a_0 is then given by

$$a_0 = \frac{S}{2E_r} \times \left(1 + k\,\frac{H}{E}\right)^{-1}$$

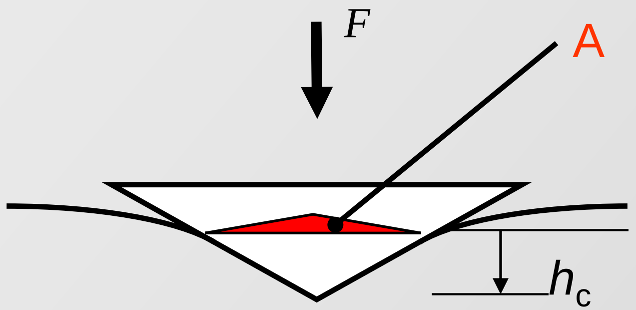
k depends on Poisson's ratio
S is the contact stiffness

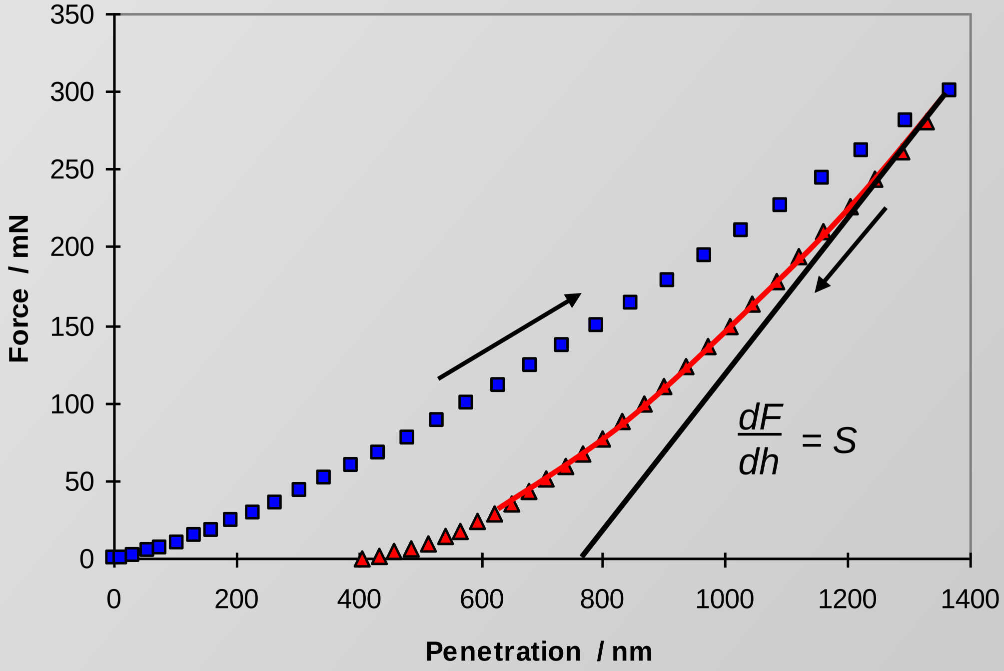


Chudoba and Jennett, J. Phys D **41** (2008) 215407

Oliver and Pharr method

Calculate material properties from unloading data (for Berkovich indenters)





Using Sneddon's result

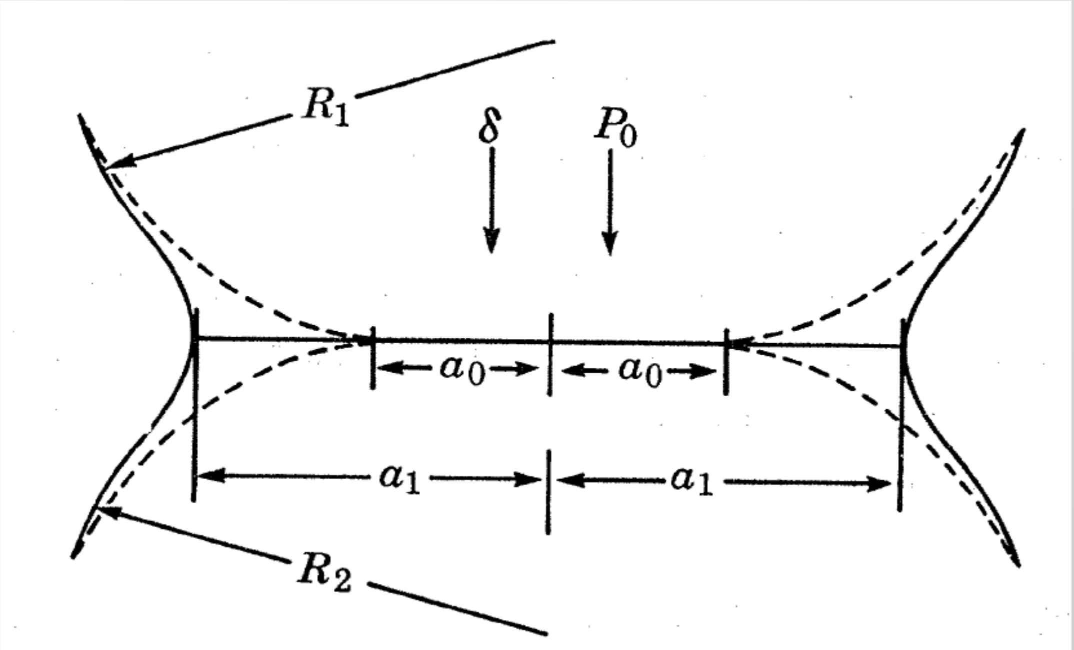
$$h_c = h_{max} - 0.75\frac{F_{max}}{S}$$

from h_c and indenter geometry get area of contact, *A*

For an ideal Berkovich indenter the contact area is a simple function of the contact depth, h_c $A = 24.56\, h_c^2$

Adhesive contact: JKR

The addition of the surface adhesive force changes the contact from asymptotic to perpendicular to the surface and the contact radius increases from a_0 to a_1



There is a finite contact radius at zero load

$$a^3 = \frac{6\gamma\pi R^{*2}}{E^*}$$

And a tensile force is needed to break the contact

$$F = -\frac{3}{2}\gamma\pi R$$

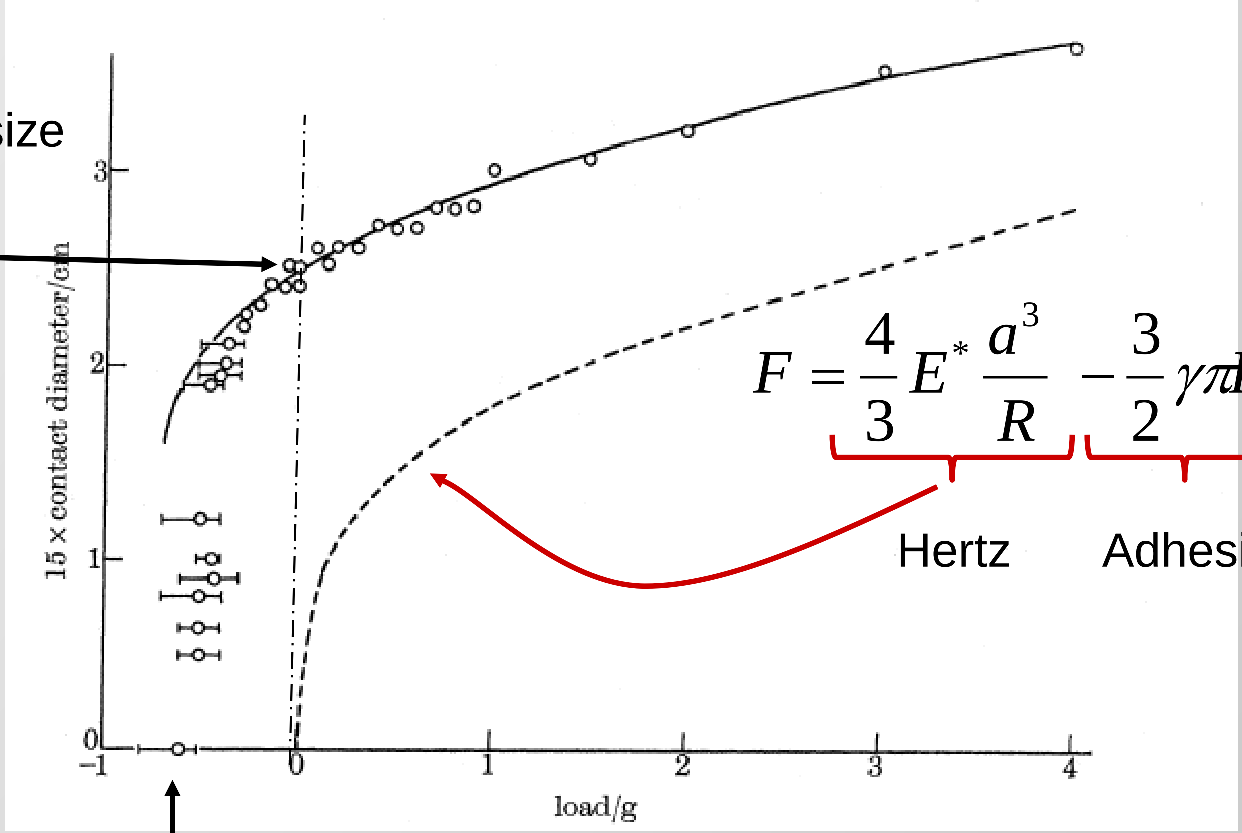
K.L.Johnson, K.Kendall, A.D.Roberts, Proc. Roy. Soc. A324, 301-313 (1971)

Adhesive contact: JKR

Hertz and JKR add to increase the total force and result in tensile 'pull-off' force

Finite contact size at zero load

$$a^3 = \frac{6\gamma\pi R^{*2}}{E^*}$$



$$F = \underbrace{\frac{4}{3}E^*\frac{a^3}{R}}_{\text{Hertz}} \underbrace{-\frac{3}{2}\gamma\pi R}_{\text{Adhesion}}$$

Hertz Adhesion

'Pull-off' force

$$F = -\frac{3}{2}\gamma\pi R$$

Summary

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- Theory that we use is based in elastic contact
- Indentation is a special case of contact mechanics
- Hertzian elastic contact mechanics is rigorous and complete
- The stress distribution beneath a contact is complex and indentations into engineering materials are not purely elastic (plasticity, time dependant, etc)
- Indentation analysis is based on elastic unloading to determine contact area
- Oliver and Pharr method most common approach for Berkovich indentation into elastic-plastic solids (metals and ceramics)
- Johnson – Kendal – Roberts for adhesive contact